1972

A Lanchester-based model for investigation of tactical decision rules for two-stage ambushes.

Metkarunchit, Treepet.

Monterey, California. Naval Postgraduate School

A LANCHESTER-BASED MODEL FOR INVESTIGATION OF TACTICAL DECISION RULES FOR TWO-STAGE AMBUSHES

Treepet Metkarunchit

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

A Lanchester-Based Model for Investigation
of Tactical Decision Rules
for Two-Stage Ambushes

by

Treepet Metkarunchit

Thesis Advisor: G. F. Lindsay

September 1972

Approved for public release; distribution unlimited.

A Lanchester-Based Model for Investigation
of
Tactical Decision Rules for Two-Stage Ambushes

by

Treepet Metkarunchit
First Lieutenant, Royal Thai Army
B.S., Chulachomklao Royal Military Academy, 1967

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
September 1972

ABSTRACT

In order to analyze ambush tactics of small units, a generalization of Lanchester Theory of Combat to a set of Lanchester-type equations modeling small force guerilla engagements is used. The tactic investigated is that of the double ambush, wherein the ambushing force is divided to stage two sequential ambushes. The withdrawal of the first ambushing force is assumed to lead a pursuing opponent into the second ambush. Ambusher force allocation, and decision rules for withdrawal in the first ambush were varied and second ambush survivors were used as a measure effectiveness. Repeated computer solutions of the Lanchester-based model permitted general conclusions about certain tactics in this ambush situation. Suggested extension to the study are also discussed.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

I. SUMMARY

Ambushes appear to be a key element in the new face of war. The tactics studied in this thesis is use an ambush to lure the enemy into another ambush. This kind of tactic can be executed by dividing the ambusher force into two forces. The first force acts as the bait to induce the enemy to the second ambush. The problem arises when decisions must be made concerning the allocation of ambusher forces between the two ambushers and establishment of ambusher withdrawal time for both engagements.

Models based on Lanchester Equation of Combat which analytically describe the first and second engagement relationship provide a planning model to assist the decision-maker in solving his problem. The combat attrition model may also be used to determine the first-ambusher force size necessary for a successful operations.

In order to analyze various tactical combinations and situations, the attrition model was computerized to permit numerical solutions of a small unit engagement. For each battle investigated, a first ambusher force size and minimum allowable percentage of survivors were varied. The computer output from 20 different battles was analyzed for tactical implications rather than specific numerical answers. This generated battle data from an ambusher of force size twelve and ambushed force of size thirty implied that a

first ambusher force of size two with ambush duration of 0.5 minutes, in combination with withdrawal time of 5.0 minutes of second ambusher force, produced the highest degree of success for the situations examined.

The result of the study showed that this tactic of ambush can be modelled and provide a planning model permitting a reasonably general simulation of the first and second engagements.

II. INTRODUCTION

An ambush is an unexpected attack on an enemy, made from a hidden place. Ambushes are one of the older tactics in the long history of war. In recent times, ambushes have become a widely used technique in Guerrilla Warfare in which many excellent examples of the concept of ambush may be found.

Ambushes are one of the most effective measures for inflicting personnel casualties on the enemy. A recent example was large scale Viet Cong ambush of a Vietnamese Marine battalion which was moving from Hue to Dong Ha by motorized convoy. The action was a classical example of an ambush and resulted in 137 Vietnamese Marine casualties [8].

The purpose of this thesis is to develop a planning model by mathematically describing a particular ambush consisting of two sequential engagements. The ambusher force is divided into two forces. The first force, after the first engagement, will lure the enemy to follow into fire from a second prepared ambush position. In the second engagement, the second force may spring the ambush and then withdraw at some appropriate time with small parties left behind to cover the withdrawal. A computer program written to solve the ambush equations is used to explore effects on engagement outcomes by the allocation of ambusher personnel

between the two ambushes and the decision rules used for withdrawal from the first ambush.

III. AMBUSHES

"A few days before Christmas, 1961, a young American soldier, Sp4 James T. Davis of Livingston, Tennessee, was riding with ten Vietnamese soldiers in a big 'dence and a half' truck a dozen or so miles from Saigon. The road was good, and there was no reason to worry about anything in particular. Without warning, a huge blast erupted from a mound of earth at the side of the road, smashing the truck like a toy, and spewing bloody troops in all directions. Davis was alive and mad, and his carbine had been loaded. He managed to empty two magazines of cartridges at the attackers. But the men in the bushes had machine guns. And so, on December 22, 1961, Specialist Davis became the first American serviceman to die in combat with Viet Cong. It is in keeping with the character of this war that Davis died in an ambush [2]."

Ambushes are used widely in guerilla warfare, and have been a favorite tactic of small and poorly armed societies against great powers. It has been written in How Castro Won by Dickey Chapelle that

> "---during the early months of the fighting, the only military tactic used by the rebels was to ambush small government patrols for their weapons. As the patrols grew larger, the rebelde underground furnished mines, and Fidelistas were able to turn back several punitive thrusts made at them in the mountains by ringing their strongholds with the mines [6]."

> "An ambush is a surprise attack, by a force lying in wait, upon a moving or temporarily halted enemy.

It is usually a brief encounter and does not require the capture and holding of ground [1]."

The success of an ambush will depend upon complete surprise in the attack. This can be achieved by a high degree of discipline, especially with respect to fire control, and the ability to break off the attack [13]. In order to disperse rapidly and respond to leadership, a high level of training is also needed. Without intelligence, the ambusher force cannot effectively plan and prepare an ambush. A force can ambush effectively by having the following, an extensive intelligence screen, information about opponent movements coupled with intimate knowledge of the terrain, and inherent mobility [13]. Most important is the spirit, tenacity, courage and second training of the individual ambushers. The conduct of the ambush showed that the ambushers acted on timely, accurate information which was coordinated into a detailed plan [13].

To some people, the ambush seems an inferior and unfair form of warfare employed by cowards. It may be all of these things, but in any fighting, the goal is victory regardless of any rules of war.

An ambush is a tactic that enables the weak to attack and destroy the strong, particularly when used by highly skilled military tacticians. Ambush tactics vary with the type of target and proximity of ambushed force to supporting units. Most ambushes are squad-sized shoot-and-run actions [12].

A deliberate ambush is one which is planned and executed as a separate operation [1]. The deliberate ambush tactic which is modeled in this thesis, is an operation luring an opponent, called ambushed force, into a second prepared ambush position. This kind of tactic has been sophisticated to such a degree that the primary ambush is not the main objective, but itself becomes the bait for a second and much larger ambush. The ambusher force divides itself into two forces. The allocation of personnel to each force is a big problem. Usually, the second force is larger than the first one and should be large enough to inflict heavy casualties on its opponents in a short period of time. On the other hand, the first force must not be too small to fail its operation.

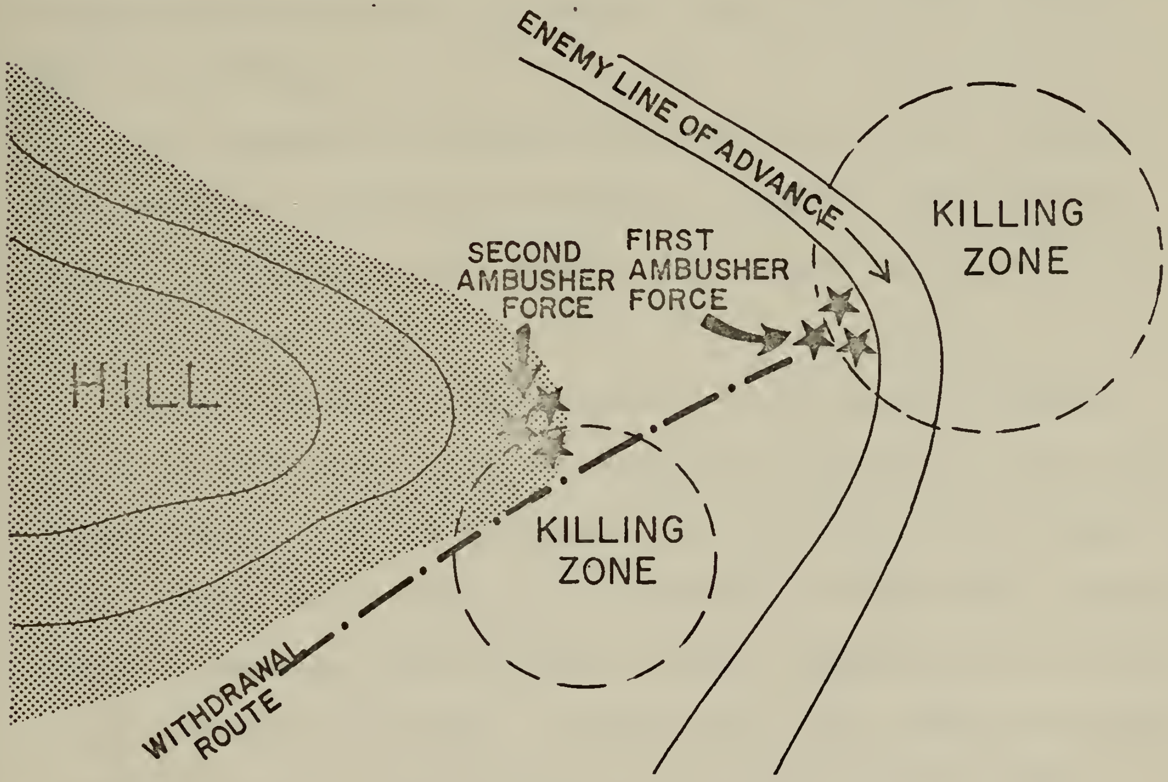


Figure 1. A possible ambush site.

The first ambusher force can be placed at known enemy route as shown in Figure 1. After springing its ambush and engaging the enemy for a short period of time, the first ambusher force must withdraw along the selected route. The time to begin withdrawal from the first engagement is important because the first ambusher force is very small in comparison to the opponent. Thus the time of the first engagement should not be too long in order that the first ambusher force will not be destroyed before withdrawal. In this thesis, it is assumed that the number of ambusher casualties and the length of engagement are the decision factors which determine the time at which withdrawal begins. The second ambusher force sets itself up in ambush not very far from the first force, and waits for its opponent. When its opponent appears in killing area, the second ambush will be sprung.

"The ambushers can make the full use of the surprise and temporary confusion achieved to inflict the maximum amount of damage on the enemy, and at the same time ensure a clean break [1]."

Often the time of withdrawal by the second ambusher force is dependent upon the specific objective of the ambusher force, and is not based on a predetermined engagement length as is done in this thesis. Frequently, however, radio communications are good and the ambushed force will call for help by radio. In this case, the ambushers must withdraw prior to arrival of reaction forces and thus

basing withdrawal time on a predetermined engagement length seems appropriate.

In the following chapter a mathematical model of an ambush will be developed, in which two successive ambushes are staged and force allocation and withdrawal decision rules are treated as decision variables.

IV. MATHEMATICS OF AMBUSHES

The development of the mathematical model of ambushes begins with two basic Lanchester equations of combat [5]. The first is the linear law formulation which can be derived from the case where the attrition caused by one force of size n per unit of time, $\frac{dn}{dt}$, is directly proportional to the numerical strength, m, of the other force, and vice versa. The linear law is applicable where the two forces are concealed from each other so that area fire is used by both. The following are the attrition rates for the two forces:

$$\frac{dm}{dt} = -k_n \, mn \tag{1}$$

$$\frac{dn}{dt} = -k_m \, mn \tag{2}$$

where k_n and k_m are coefficients reflecting the ability of each force to kill its opponents, t is the time since the beginning of the battle.

The solution to these equations yields the result that when two sides are reduced to zero at the same time,

$$k_m \, m_o = k_n \, n_o \tag{3}$$

where m_o and n_o are the initial numerical strengths of m and n respectively.

A second set of equations applies to the case where combatants attack each other in such manner that units in each force may take any opponent unit under fire and may

shift fire to another enemy unit when desired. Here, the attrition rates for the two forces can be expressed as

$$\frac{dm}{dt} = -k_n n \qquad (4)$$

$$\frac{dn}{dt} = -k_m m \qquad (5)$$

The solution to these equations yields the result that when opposing sides will reach zero strength at the same time,

$$k_m (m_o)^2 = k_n (n_o)^2 \qquad (6)$$

In an effort to develop a model for ambush, Deitchman proposed a mixed linear-square law for ambushes [10]. Deitchman's Ambush Model assumes that all members of each side are armed with the same type of weapon and ambushers will be firing point fire, while ambushed force will be using area fire. The attrition rates of two forces are

$$\frac{dm}{dt} = -k_n n \qquad (7)$$

$$\frac{dn}{dt} = -k_m mn \qquad (8)$$

In this case k_n is a point-fire kill coefficient and k_m is area fire kill coefficient. Deitchman defined the ambusher kill coefficient k_n as

$$k_n = r_n p_n \qquad (9)$$

where r_n is the rate of fire of each of n's weapons, and p_n is the single shot kill probability of each n's weapons in point fire. Similarly, the ambushed force's attrition coefficient is defined as

$$k_m = r_m \frac{A_e}{A_n} \tag{10}$$

where r_m is the rate of fire of ambushed force, Ae is the area of a target presented by ambushers exposed to fire and An is the total area in which ambushers are concealed. That is, the single shot hit probability of ambushed force's weapons using area fire is the ratio $\frac{A_e}{A_n}$.

The solution to the equations (7) and (8) gives a mixed law, and from this it may be shown that the condition for both sides to be reduced to zero at the same time is

$$2k_n n_o = k_m (m_o)^2 \tag{11}$$

where n_o and m_o are the initial strengths of the ambusher and ambushed forces, respectively.

Deitchman's ambush model allows ambushers to take advantage of concealment, and can be considered as a reasonable model for the battle. But, there is some deficiency in equation (7) and (8) since the attrition coefficients k_n and k_m do not vary with time. This means that ambushed force cannot take cover, nor can it improve its aiming capability during the battle.

Schaffer has developed an ambush model which does take into account changes over time during an engagement [7]. Schaffer's model is sufficiently general to represent a wide variety of ambush-engagement situations. His model explicitly treats time variance of weapons efficiency coefficients, morale and discipline factors, and supporting weapons. Schaffer's ambush model for the attrition of an

ambushed force of size m, is expressed as

$$\frac{dm}{dt} = -(1-b_m)k_n(t)n - c_m(\frac{n}{m} - 1)^2$$

$$-(1-b_m)\sum_i E_i(t,m)w_i(t) \qquad (12)$$

where b_m and c_m are constants associated with troop morale and discipline of ambushed force. The ambusher force of size n is assumed to have a small arms weapon-efficiency coefficient

$$k_n(t) = \frac{r_n A_T(t) p_{k_n}}{2\pi\sigma_n^2} \qquad (13)$$

where r_n is ambusher rate of fire,

p_{k_n} is the probability of kill given a hit on target,

σ_n is the single-shot radial dispersion of fire,

$A_T(t)$ is the area of ambushed force exposed to fire over time, expressed as

$$A_T(t) = \frac{A_{T_\infty}}{(1-e^{-\alpha t-\beta})} \qquad (14)$$

In equation (14), A_{T_∞} is the steady-state value of minimum presented area by ambushed force who have had time to find maximum cover, α, reflects the speed with which ambushed force attains maximum cover, and β determines their target size when the ambush begins at t=0. The term $(1-b_m)\sum_i E_i(t,m)w_i(t)$, added to equation (12), represents the casualties produced by the supporting weapons of the ambusher, where

E_i are the weapons efficiencies of the ambusher's i types of supporting weapons over time, and

w_i are the supporting-weapon strengths of type i over time.

Ambusher attrition is expressed as

$$\frac{dn}{dt} = -k_m(n,t)m - c_n(t)\left(\frac{m}{n}-1\right)^2 - \sum_j E_j(t,n)w_j(t) \tag{15}$$

In equation (15), $k_m(n,t)$ is time dependent and reflects the ambushed force's change from area fire to point fire by an exponential function

$$k_m(n,t) = k''(1-e^{-\gamma t}) + k'ne^{-\gamma t} \tag{16}$$

where γ determines the rate of shifting from area fire to point fire, k' and k" are ambushed force kill effectiveness when using area fire and point fire, respectively. It can be seen that, if t=0, equation (16) reduces to

$$k_m = k'n = r_m \left(\frac{A_e}{A_n}\right) p_{k_m} n \tag{17}$$

and if $t=\infty$, it reduces to

$$k_m = k'' = r_m \left(\frac{A_e}{2\pi\sigma_m^2}\right) p_{k_m} \tag{18}$$

where r_m is ambushed force rate of fire,

σ_m is radial dispersion of a simple ambushed force round, and

p_{k_m} is the probability of kill given a hit on target.

In equation (15), $c_n(t)$ and $(\frac{m}{n}-1)^2$ represent the act of breaking off an engagement, and coefficient $c_n(t)$ is defined as

$$c_n(t) = |c_n| H(t - t_c)\, H(\frac{m}{n} - 1) \tag{19}$$

where H is the unit step function, t_c is the time required for the discipline of an ambusher to deteriorate to the point where he may not fight, and c_n reflects the training and motivation of ambushers. It can be seen that $c_n(t)$ is a positive quantity when $t > t_c$ and $\frac{m}{n} > 1$, and is zero otherwise. The term $\sum_j E_j(t,n) w_j(t)$ added to the equation (15) represents the ambushed force coefficients of j types of support weapons where w_j is the number of each type j employed.

In the following development of the ambush model, Schaffer's equations will be used and modified as necessary to reflect the conditions existing in an ambush. It is assumed that no desertions or surrenders take place on either side.

A. AMBUSHER MODEL

The ambusher force of size n is divided such that n_1 and n_2 are the number of ambushers assigned to the first and second forces respectively. In the first engagement, a first ambusher force sets an ambush without support weapons, and breaks off the engagement quickly without using a rear of guard element. Thus, the coefficient $c_n(t)$ in equation (15) can be set at zero. In a short period

time of engagement, the ambushed force cannot be supported by any reaction force. That is, the term $\sum_j E_j(t,n)w_j(t)$ can also be set at zero. Now equation (15) is reduced and can be considered as the first ambusher force attrition rate, i.e.,

$$\frac{dn_1}{dt} = -k_m(n,t) \tag{20}$$

Similarly, in the second engagement, second ambusher force sets an ambush without supporting fire and makes a deliberate decision to commence a gradual withdrawal before the arrival of reaction forces. Thus, the term $\sum_j E_j(t,n)w_j(t)$ can be set at zero. The second ambusher force attrition can be expressed as

$$\frac{dn_2}{dt} = -k_m(n,t) - c_n(t)\left(\frac{m}{n} - 1\right)^2 \tag{21}$$

B. AMBUSHED FORCE MODEL

Assuming that no desertions or surrenders take place in the ambushed force and no support weapons are used by the ambusher forces, the coefficient b_m and c_m and the term $\sum_i E_i(t,m)w_i(t)$ can be set at zero. From equation (12), the ambushed force attrition then becomes

$$\frac{dm}{dt} = -k_n(t)n \tag{22}$$

The equations (20), (21) and (22) are used as tactical ambush models in the following chapters.

V. NUMERICAL SOLUTIONS TO THE AMBUSH MODEL

In the previous chapter, a model for two-stage sequential ambushes was developed. The model contains parameters for which numerical estimates are needed. These parameters will be discussed and estimates suggested for use in the ambush model as approximations of actual combat values.

A. INVESTIGATION OF PARAMETERS

For parameters in equations (13), (14), (16) and (19) the following values have been suggested by Burnell [11]:

$$A_{T_\infty} = 1.68 \text{ ft}^2$$

$$\alpha = 0.572$$

$$\beta = 0.4$$

$$p_{k_n} = 0.7$$

$$\sigma_n = \sigma_m = 10 \text{ mil}$$

$$\gamma = 0.516 \text{ min}^{-1}, \text{ and}$$

$$c_n = 1.0$$

This thesis will also follow Schaffer in setting $k' = 2.4 \times 10^{-5}$, $t_c = 10$ min, and $p_{k_m} = 0.5$ [8]. A maximum rate of fire is 40 rounds per minute, which is the sustained rate of fire of the M-14 rifle [4]. Other studies have suggested that the rate of fire, r_n, should be obtained by multiplying 0.5 to the theoretical rate of fire [11]. Thus, the rate of fire should be 20 rounds per minute. Also, r_m is assumed to be equal to r_n.

An ambusher in a prepared defensive area may be considered a circular target and a radius of 4.95 inches is used for the radius of the circular target area [9]. This implies that target area presented by ambusher, Ae, is 0.54 ft^2. With these parameter values, the ambushed force kill effectiveness by using point fire, k", can be obtained by solving equation (18).

B. VARIABLES OF BATTLE SIMULATION

The initial ambusher force size is set at twelve men. The first ambusher force of size n_1 will be varied from 2 to 6 men, such values are chosen as candidate values for this decision variable. The second ambusher force size, n_2, depends upon the first ambusher force size, i.e., it varies from 6 to 12 men. The ambushed force size, m, is set at thirty men for all engagement simulations. The maximum length of first engagement is set at 3.0 minutes.

The second ambush duration depends upon withdrawal time of second ambusher force. This thesis will follow Harden in assuming an arrival of a reaction force as early as 5 minutes, in this case after the beginning of the second ambush [12]. Thus, the second ambusher force will break off an engagement at time 5.0 minutes.

C. AMBUSH MODEL RESULTS

A computer program (in Appendix A) used the parameter values of section A and variable values of section B to generate a solution to the equations (20), (21) and (22).

In the first engagement, the logic of the program was to find solutions to the ambush equations up to either the withdrawal time of the first ambusher force or a time constraint based upon the minimum number of survivors of the first ambusher force. The latter was varied, with values of 50, 60, 70 and 80 percent used. In the second engagement, the equations were solved up to the time when either side's forces were reduced to zero. The program printed out the time required to reduce one side to zero, and the numerical strength of the winning side at the end of the engagement.

The result of computer battle simulations are displayed in Table 1 and 2. Table 1 shows the effect of changes in the minimum allowable percentage of survivors and the initial force size for the first ambusher force. For each second engagement simulation, the result that ambusher force wins, can be examined.

The total maximum number of survivors, given that ambusher force win, is seven. The maximum number of survivors of the second ambusher force is six; this is achieved by using a first ambusher force of size two and setting the allowable number of survivors at 50 percent. This means that if initial size of the ambushed force is 30 and the total size of the ambusher force is 12, then the first ambusher force size should be two. On the other hand, the number of second ambushers should be 10, when the first ambusher force breaks off an engagement at time 0.5 minutes

TABLE 1

n_1	Min. Percent of Survivors	First Engagement		Second Engagement		$x_1 + x_2$	Winner
		Time (min)	x_1	Time (min)	x_2		
2	50	0.5	1	7.0	6	7	Ambusher Force
	60	0.0	2	7.0	5	7	"
	70	0.0	2	7.0	5	7	"
	80	0.0	2	7.0	5	7	"
3	50	1.0	2	7.0	5	7	Ambusher Force
	60	1.0	2	7.0	5	7	"
	70	0.0	3	10.5	2	5	"
	80	0.0	3	10.5	2	5	"
4	50	1.5	2	7.5	3	5	Ambusher Force
	60	1.0	3	9.0	2	5	"
	70	1.0	3	9.0	2	5	"
	80	0.0	4	7.0	0	4	Ambushed Force
5	50	2.0	3	9.5	2	5	Ambusher Force
	60	1.5	3	10.5	0	3	Ambushed Force
	70	1.0	4	8.5	0	4	"
	80	0.5	4	6.5	0	4	"
6	50	2.5	3	10.5	1	4	Ambusher Force
	60	2.0	4	8.5	0	4	Ambushed Force
	70	1.0	5	6.0	0	5	"
	80	1.0	5	6.0	0	5	"

n_1 = The first ambusher force size
x_1 = The number of survivors of first ambusher force
x_2 = The number of survivors of second ambusher force
$x_1 + x_2$ = Total number of survivors

TABLE 2

The number of survivors of both sides in the second engagement at time 3.0, 4.0, and 5.0 minutes.

n_1	Minimum Percent of Survivors	x_1	Ambush duration (min) 3.0				4.0				5.0			
			x_2	x_1+x_2	y	z	x_2	x_1+x_2	y	z	x_2	x_1+x_2	y	z
2	50	1	7	8	9	0.89	7	8	5	1.60	6	7	2	3.50
	60	2	7	9	11	0.82	6	8	8	1.00	6	8	5	1.60
	70	2	7	9	11	0.82	6	8	8	1.00	6	8	5	1.60
	80	2	7	9	11	0.82	6	8	8	1.00	6	8	5	1.60
3	50	2	6	8	10	0.80	6	8	7	1.14	5	7	4	1.75
	60	2	6	8	10	0.80	6	8	7	1.14	5	7	4	1.75
	70	3	6	9	13	0.69	5	8	10	0.80	4	7	8	0.88
	80	3	6	9	13	0.69	5	8	10	0.80	4	7	8	0.88
4	50	2	5	7	10	0.70	5	7	7	1.0	4	6	5	1.20
	60	3	5	8	11	0.73	4	7	8	0.88	4	7	6	1.17
	70	3	5	8	11	0.73	4	7	8	0.88	4	7	6	1.17
	80	4	5	9	15	0.60	4	8	13	0.62	2	6	11	0.55
5	50	3	5	8	10	0.80	4	7	7	1.00	3	6	6	1.00
	60	3	4	7	11	0.64	4	7	9	0.78	3	6	7	0.86
	70	4	4	8	12	0.67	3	7	10	0.70	2	6	8	0.75
	80	4	4	8	14	0.57	3	7	12	0.58	2	6	11	0.55
6	50	3	4	7	9	0.76	3	6	7	0.86	3	6	5	1.20
	60	4	4	8	10	0.80	3	7	8	0.88	2	6	6	1.00
	70	5	3	8	13	0.62	2	7	11	0.64	1	6	10	0.60
	80	5	3	8	13	0.62	2	7	11	0.64	1	6	10	0.60

n_1 = The first ambusher force size
x_1 = The number of survivors of the first ambusher force
x_2 = The number of survivors of the second ambusher force
y = The number of survivors of the ambushed force
$z = (x_1 + x_2)/y$

to make sure that second ambusher force can win in the second engagement. In this thesis, the number of casualties on both sides is measured as expected values. If the first ambusher force size is one and he is killed before withdrawal, then the operations has failed because nobody is left for luring the ambushed force into the second ambush.

Assuming a possible arrival reaction force five minutes after the start of the second ambusher means that, from the ambusher's point of view, second ambush duration must be less than or equal to five. Table 2 shows the result of second engagement simulation when the second ambusher force breaks off at time 3.0, 4.0, and 5.0 minutes, respectively. The maximum ratio of number of survivors between the total ambusher force and ambushed force is 3.5. This supports the result from Table 1 that the size of first ambusher force should be 2 and the length of an engagement should be 0.5 minutes, with the minimum allowable number of survivors 50 percent. Thus in the second engagement, the withdrawal time of 5.0 minutes of the second ambusher force is reasonable.

VI. CONCLUSIONS AND EXTENSIONS

It has been shown that the ambusher force and ambushed force relationship in the two-stage ambushes can be modeled, and these ambush equations are felt to contain enough variables to allow a realistic portrayal of many ambush situations. This is a type of ambush in which the ambusher force does not need overwhelming superiority of numbers to win. From Table 1, it was seen that twelve ambushers stand a reasonable chance of defeating 30 ambushed troops with small losses to themselves using a kind of ambush in which ambushers set up decoys in planning their ambushes, and the second ambush is the main objective. Actually, the second ambusher force can inflict effectively the maximum amount of damage on the enemy by using grenades, mines, and perfect camouflage [2,1]. Thus, the ambushed force model, equation (22), showed be changed. As an extension of this thesis, it is suggested that further application of the model could be made by modifying the small arms weapon-efficiency coefficient, $k_n(t)$ to include a mixture of weapons.

This thesis attempted to use reasonable values of the parameters of the model in order to assure an adequate representation of this situation involving a small-unit infantry battle. The ambushers who fire could be expected to fire at a high rate initially with a decline to a slower

rate to prevent overheating of their rifles [4]. Usually, automatic guns are used initially at a high rate when an ambush is sprung. Burnell suggested that during the first minute when the high value of r_n is being used [11], the coefficient $k_n(t)$ should be

$$k_n(t) = r_n \left(\frac{A_v}{A_m}\right) p_{k_n} m$$

where A_v is the exposed area of one ambushed soldier, and A_m is the area occupied by the ambushed unit. Then, when r_n is given a lower long term value, $k_n(t)$ should be

$$k_n(t) = \frac{r_n A_T(t) \; p_{k_n}}{2\pi\sigma_n^2}$$

In this thesis, it is not implied that present tactics can or should be changed based on the results of this study or other present models of combat engagements. However, it is strongly felt that modeling of military tactics could produce interesting results which might aid the combat commander in his decision making process.

APPENDIX A

COMPUTER PROGRAM USED TO SOLVE THE AMBUSH EQUATIONS

This computer program was written in FORTAN IV language for IBM360 series computer. Preceding the main program is a list of variables and their definitions. Comment cards have been used where necessary to explain the program. Sample data cards and a sample of the output are included.

```
C     THIS COMPUTER  PROGRAM CONSISTS OF TWO PARTS , ONE TO DETERMINE
C     THE OUTCOME OF THE FIRST AMBUSHER FORCE VS AMBUSHED FORCE ,
C     AND THE OTHER TO DETERMINE THE OUTCOME OF THE SECOND AMBUSHER
C     FORCE VS THE REMAINDER OF AMBUSHED FORCE
C
C
C
C     THE SYMBOLS USED IN THE PROGRAM ARE DEFINED AS FOLLOWS
C
C
C
C     XN    = TOTAL FORCE OF AMBUSHER
C     XN1   = FIRST INITIAL FORCE OF AMBUSHER
C     XN2   = SECOND INITIAL FORCE OF AMBUSHER
C     XM    = INITIAL AMBUSHED FORCE
C     RN    = AVERAGE RATE OF RIFLE FIRE
C     AT    = STEADY-STATE VALUE OF THE DEFENCIVE-COVER
C     ALPHA = CONSTANT WHICH DETERMINES THE RATE AT WHICH AMBUSHED FORCE
C             CAN ATTAIN COVER
C     BETA  = CONSTANT WHICH DETERMINES THE PRESENTED AREA OF THE
C             INDIVIDUAL IN THE AMBUSHED FORCE AT THE BEGINNING OF THE
C             AMBUSH
C     ZIGMA = SINGLE-SHOT RADIAL DISPERSION OF FIRE
C     XKMT  = AMBUSHED FORCE SMALL-ARMS WEAPON-EFFICIENCY COEFFICIENT
C     XK    = AMBUSHED FORCE WEAPONS EFFICIENCY COEFFICIENT FOR AREA
C             FIRE
C     XKK   = AMBUSHED FORCE WEAPONS EFFICIENCY COEFFICIENT FOR AIMED
C             FIRE
C     GAMMA = CONSTANT WHICH DETERMINES THE RATE AT WHICH AMBUSHED FORCE
C             SHIFTS FROM AREA TO AIMED FIRE
C     CNT   = CONSTANT REFLECT THE TRAINING AND MOTIVATION OF AMBUSHER
C     CN    = CONSTANT WHICH DETERMINES THE RATE AT WHICH AMBUSHER
C             WITHDRAW
C     TC    = TIME AT WHICH AMBUSHER MAY BEGIN TO WITHDRAW IF
C             OUTNUMBERED
C     PHK   = SINGLE-HIT KILL PROBABILITY
C     TD    = MAXIMUM WITHDRAWAL TIME  OF THE FIRST AMBUSHER FORCE
C     P,Q   = TIME INCREMENT USED IN RUNGE-KUTTA-GILL INTEGRATION
C             ROUTINE
C     PC    = AMBUSHER'S MINIMUM PERCENT SURVIVORS
C
C
C
C
      COMMON /COM1/ Q,TC /COM2/ APHK,ALPHA,BETA /COM3/ XK,XKK,GAMMA/COM4
     1/ CN /COM5/ P,TD
C
C
C     INPUT DATA
C
C
```

```
      TD     = 3.0
      RN     = 20.0
      AT     = 1.68
      BETA   = 0.4
      ALPHA  = 0.572
      GAMMA  = 0.516
      ZIGMA  = 2.952
      XK     = 0.000024
      XKK    = 0.09761
      CN     = 1.0
      TC     = 10.0
      Q      = 0.5
      P      = 0.5
      PHK    = 0.7
C
C
      A      = RN*AT/(2.0*3.1416*ZIGMA*ZIGMA)
      APHK   = A*PHK
 1110 READ (5,8) XN, XN1,XM
    8 FORMAT (6X,3(F5.1,2X))
      IF (XN.EQ.0.0) GO TO 1113
      M      = XM
      N1     = XN1
      N      = XN
      PC     = 40.0
 1111 PC     = PC + 10.0
      IPC    = PC
      WRITE (6,9) N,M
    9 FORMAT ('1',//,23X,60('*'),//,37X,'TOTAL AMBUSHER FORCE = ',I3,//,
     137X,'TOTAL AMBUSHED FORCE = ',I3,//,23X,60('*'))
      WRITE (6,11) N1,M,TD,IPC
   11 FORMAT (//,41X,'FIRST ENGAGEMENT',//,28X,'INITIAL AMBUSHER FORCE .
     2.......... = ',I3,//,28X,'INITIAL AMBUSHED FORCE ........... = ',I
     33,//,28X,'AMBUSHERS  MAX. TIME OF ENGAGEMENT = ',F3.1,//,28X,'AMBU
     4SHERS  MIN. PERCENT SURVIVORS  = ',I3)
      WRITE (6,12)
   12 FORMAT (//,27X,'TIME(MIN)',3X,'AMBUSHER FORCE',3X,'AMBUSHED FORCE'
     5)
C
C
C     THE OUTCOME OF THE FIRST AMBUSHER FORCE VS AMBUSHED FORCE
C
C
      CALL ACT1 (XM,XN1,MD,ND,PC)
      N2     = XN-XN1
      WRITE (6,13) N2,MD
   13 FORMAT (////,41X,'SECOND ENGAGEMENT',//,29X,'INITIAL AMBUSHER FORC
     1E ......... = ',I3,//,29X,'INITIAL AMBUSHED FORCE ......... = ',I3
```

```
      2)
       WRITE (6,12)
       XM1    = MD
       XN2    = N2
C
C
C      THE OUTCOME OF THE SECOND AMBUSHER FORCE VS THE REMAINDER OF
C      AMBUSHED FORCE
C
C
       CALL ACT2 (XM1,XN2,KD,LD)
       JN     = LD + ND
       IF (KD.GT.LD) GO TO 503
       WRITE (6,14) JN
    14 FORMAT (////,33X,'AMBUSHER WINS SECOND ENGAGEMENT',//,23X,'TOTAL A
      6MBUSHER SURVIVORS (BOTH ENGAGEMENT) = ',I3)
       GO TO 504
   503 WRITE (6,16)
    16 FORMAT (////,30X,'AMBUSHED FORCE WINS SECOND ENGAGEMENT')
   504 IF (PC-80.0) 1111,1110,1110
  1113 STOP
       END
C
C
C      SUBROUTINES ACT1 AND ACT2 USE A RUNGE-KUTTA-GILL 4-TH ORDER
C      NUMERICAL INTEGRATION TECHNIQUE TO EVALUATE THE THE SIMULTANEOUS
C      DIFFERENTIAL EQUATIONS
C
C
       SUBROUTINE ACT1 (XMX,XN1X,MY,NY,PY)
       COMMON /COM5/ P,TD
       T      = 0.0
       DMY    = XMX
       DNY    = XN1X
       IF (T-0.0) 104,104,101
   101 IF (T-TD) 103,121,121
   103 F1     = P*AA(DNY,T)
       G1     = P*BB(DMY,DNY,T)
       F2     = P*AA(DNY+G1/2.0,T+P/2.0)
       G2     = P*BB(DMY+F1/2.0,DNY+G1/2.0,T+P/2.0)
       F3     = P*AA(DNY+G2/2.0,T+P/2.0)
       G3     = P*BB(DMY+F2/2.0,DNY+G2/2.0,T+P/2.0)
       F4     = P*AA(DNY+G3,T+P)
       G4     = P*BB(DMY+F3,DNY+G3,T+P)
       DMY    = DMY+(F1+2.0*F2+2.0*F3+F4)/6.0
       DNY    = DNY+(G1+2.0*G2+2.0*G3+G4)/6.0
```

```
      T      = T+P
  104 LY     = DNY
      YNY    = LY
      PR     = 100.0*YNY/XN1X
      IF (PY-PR) 105,105,121
  105 MY     = DMY
      NY     = DNY
      WRITE (6,15) T,NY,MY
   15 FORMAT (/,28X,F6.2,9X,I3,15X,I3)
      IF  (PR.GT.PY) GO TO 101
  121 RETURN
      END
C
      SUBROUTINE ACT2 (XM1X,XN2X,MY,NY)
      COMMON /COM1/ Q,TC
      T      = 0.0
      DMY    = XM1X
      DNY    = XN2X
      IF (T-0.0) 112,112,104
  104 IF (T-TC) 106,105,105
  105 IF (XM1X-XN2X) 106,106,107
  106 F1     = Q*AA(DNY,T)
      G1     = Q*BB(DMY,DNY,T)
      F2     = Q*AA(DNY+G1/2.0,T+Q/2.0)
      G2     = Q*BB(DMY+F1/2.0,DNY+G1/2.0,T+Q/2.0)
      F3     = Q*AA(DNY+G2/2.0,T+Q/2.0)
      G3     = Q*BB(DMY+F2/2.0,DNY+G2/2.0,T+Q/2.0)
      F4     = Q*AA(DNY+G3,T+Q)
      G4     = Q*BB(DMY+F3,DNY+G3,T+Q)
      GO TO 108
  107 F1     = Q*AA(DNY,T)
      G1     = Q*CC(DMY,XN2X,T)
      F2     = Q*AA(DNY+G1/2.0,T+Q/2.0)
      G2     = Q*CC(DMY+F1/2.0,DNY+G1/2.0,T+Q/2.0)
      F3     = Q*AA(DNY+G2/2.0,T+Q/2.0)
      G3     = Q*CC(DMY+F2/2.0,DNY+G2/2.0,T+Q/2.0)
      F4     = Q*AA(DNY+G3,T+Q)
      G4     = Q*CC(DMY+F3,DNY+G3,T+Q)
  108 DMY    = DMY+(F1+2.0*F2+2.0*F3+F4)/6.0
      DNY    = DNY+(G1+2.0*G2+2.0*G3+G4)/6.0
      T      = T +Q
      IF (DMY) 109,109,110
  109 DMY    = 0.0
      GO TO 112
  110 IF (DNY) 111,111,112
  111 DNY    = 0.0
```

```
  112 MY     = DMY
      NY     = DNY
      WRITE (6,17) T,NY,MY
   17 FORMAT (/,28X,F6.2,9X,I3,15X,I3)
      IF (MY) 114,114,113
  113 IF (NY) 114,114,104
  114 RETURN
      END
C
      FUNCTION AA(XN10,T)
      COMMON /COM2/ APHK,ALPHA,BETA
      AA     = -XN10*APHK/(1.0-EXP(-ALPHA*T-BETA))
      RETURN
      END
C
      FUNCTION BB(XMO,XN10,T)
      COMMON /COM3/ XK,XKK,GAMMA
      EX     = EXP(-GAMMA*T)
      BB     = -(XKK*(1.0-EX)+XK*XN10*EX)*XMO
      RETURN
      END
C
      FUNCTION CC(XMO,XN20,T)
      COMMON /COM3/ XK,XKK,GAMMA /COM4/ CN
      EX     = EXP(-GAMMA*T)
      CC     = -(XKK*(1.0-EX)+XK*XN20*EX)*XMO-CN*(XMO/XN20-1.0)**2
      RETURN
      END
```

SAMPLE OUTPUT

```
************************************************************
              TOTAL AMBUSHER FORCE =  12
              TOTAL AMBUSHED FORCE =  30
************************************************************

             FIRST ENGAGEMENT
     INITIAL AMBUSHER FORCE ........... =   2
     INITIAL AMBUSHED FORCE ........... =  30
     AMBUSHERS  MAX. TIME OF ENGAGEMENT = 3.0
     AMBUSHERS  MIN. PERCENT SURVIVORS  =  50

    TIME(MIN)   AMBUSHER FORCE   AMBUSHED FORCE
      0.0             2                30
      0.50            1                28
```

SECOND ENGAGEMENT

INITIAL ABUSHER FORCE .... = 10

INITIAL ABUSHED FORCE .... = 28

TIME(MIN)	AMBUSHER FORCE	AMBUSHED FORCE
0.0	10	28
0.50	9	22
1.00	9	19
1.50	9	16
2.00	8	13
2.50	8	11
3.00	7	9
3.50	7	7
4.00	7	5
4.50	7	4
5.00	6	2
5.50	6	0

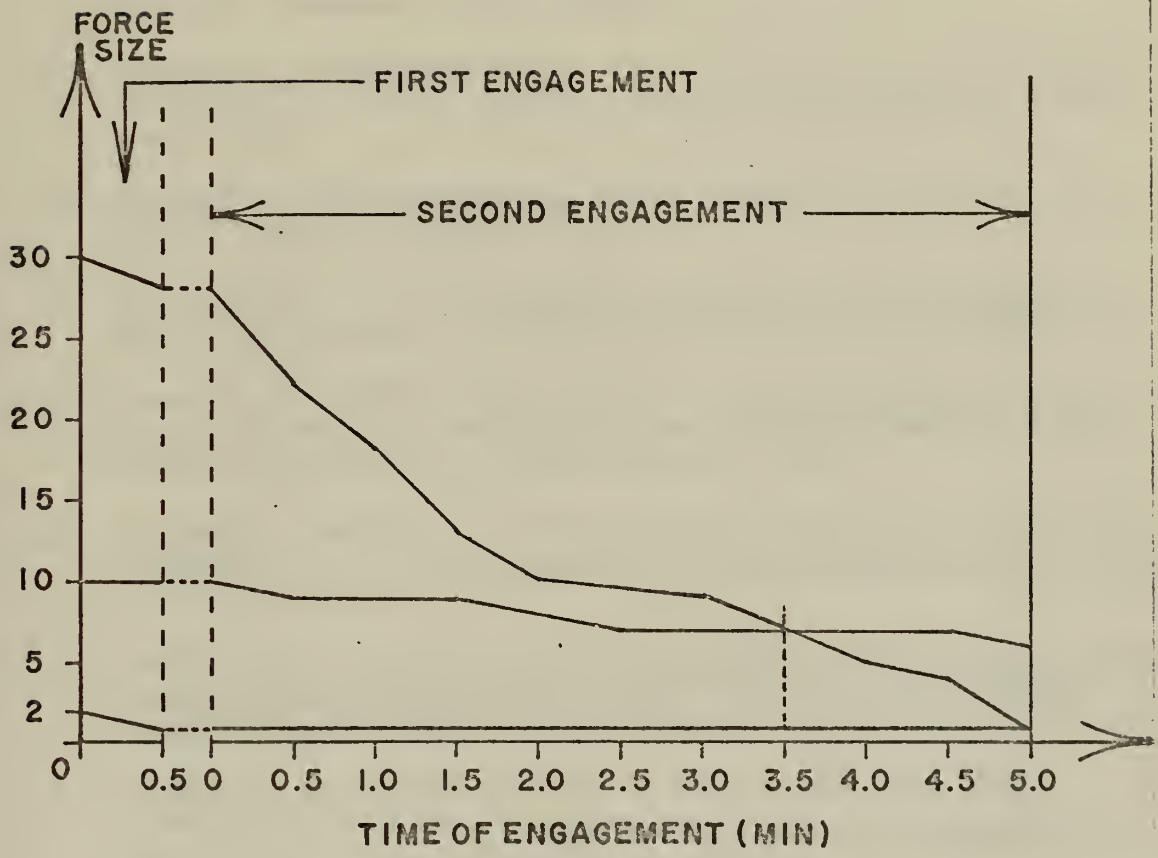
AMBUSHER WINS SECOND ENGAGEMENT

TOTAL AMBUSHER SURVIVORS (BOTH ENGAGEMENT) = 7

APPENDIX B

GRAPHS OF FORCE SIZE VS. TIME OF ENGAGEMENT

The outcome of the computer ambush model have been plotted to illustrate attrition on both sides. Figure 2 shows a plot of force size vs. time of engagement.



First ambusher force size is 2

Second ambusher force size is 10

Ambushed force is 30

The time of first engagement is 0.5 min.

The time of second engagement is 5.0 min.

Figure 2. Force size vs. time of engagement.

LIST OF REFERENCES

1. Australian Military Forces, Ambush and Counter Ambush, (Military Board, Army Headquarters, Canbera, January 2, 1966).

2. Browne, Malcolm W., The New Face of War, The Bobbs-Merrill Company, Inc., 1965.

3. Infantry Reference Data, "Characteristics of Infantry Weapons," p. 400, United States Army Infantry School, July 1968.

4. Infantry Reference Data, Fort Benning, Georgia, U. S. Army Infantry School, 1966.

5. Lanchester, F. W., Aircraft in Warfare; the Down of the Fourth Arm, Constable and Co., London, 1916.

6. Osanka, Franklin Mark, Editor, Modern Guerrilla Warfare, Fighting Communist Guerrilla Movements, 1941-1961, The Free Press of Glencoe, 1963.

7. Schaffer, Marbin B., "Lanchester Models of Guerrilla Engagements," Rand Corporation Memorandum RM-5053-ARPA, January 1967.

8. NAVMC 2614, Professional Knowledge Gained from Operational Experience in Vietnam, Headquarters, United States Marine Corps, Department of the Navy, 1965-1966.

9. Stanford Research Institute SRI 9-330, Excerpts [UNCLASSIFIED] from System Analysis of Small Arms,(u), by Barry A. Ryan, Francis A. Fulton, and others, April 1969.

10. Deitchman, S. J., Applications of Operations Research to Internal War, Army OR Symposium, Proceedings, (Army Missle Command, Redstone Arsenal, Alabama, March, 1965).

11. Burnell, Robert W., (Major, USA), Parameter Estimates for Mathematical Models of Convoy Ambushers, Master's Thesis, Naval Postgraduate School, Monterey, California, June 1968 [UNCLASSIFIED].

12. Harden, Jon David, (Lt, USN), A Planning Model for Route Security, Master's Thesis, Naval Postgraduate School, Monterey, California, 1967.

13. Kenney, J. F. C., Jr., (Major, Infantry), Counterinsurgency: One Approach to Doctrine, Master's Thesis, Fort Leavenworth, Kansas, 1964.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	2
2.	Library, Code 0212 Naval Postgraduate School Monterey, California 93940	2
3.	Asst Professor G. F. Lindsay, Code 55 Ls Department of Operations Research Naval Postgraduate School Monterey, California 93940	1
4.	1LT Treepet Metkarunchit, RTA 4 Soi Kingpet, Petburi Road Bangkok, Thailand	1
5.	Naval Postgraduate School Department of Operations Research and Administrative Sciences Monterey, California 93940	1
6.	Chief of Naval Personnel Pers 11b Department of the Navy Washington, D. C. 20370	1

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

ORIGINATING ACTIVITY *(Corporate author)* Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified
	2b. GROUP

REPORT TITLE

A Lanchester-Based Model for Investigation of Tactical Decision Rules for Two-Stage Ambushes

DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Master's Thesis, September 1972

AUTHOR(S) *(First name, middle initial, last name)*

Treepet Metkarunchit

REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
September 1972	44	13

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO.	
c.	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
d.	

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Naval Postgraduate School Monterey, California 93940

13. ABSTRACT

In order to analyze ambush tactics of small units, a generalization of Lanchester Theory of Combat to a set of Lanchester-type equations modeling small force guerilla engagements is used. The tactic investigated is that of the double ambush, wherein the ambushing force is divided to stage two sequential ambushes. The withdrawal of the first ambushing force is assumed to lead a pursuing opponent into the second ambush. Ambusher force allocation, and decision rules for withdrawal in the first ambush were varied and second ambush survivors were used as a measure effectiveness. Repeated computer solutions of the Lanchester-based model permitted general conclusions about certain tactics in this ambush situation. Suggested extension to the study are also discussed.

DD FORM 1473 1 NOV 65 (PAGE 1)

S/N 0101-807-6811

Security Classification

A-31408

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Lanchester Equations						
Ambusher Force						
Ambushed Force						
Reaction Force						